EFFECT OF DIFFUSION ON THE EVOLUTION OF A GAS BUBBLE IN AN ACOUSTIC FIELD

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The effect of diffusion on the behavior of a gas bubble in the field of an acoustic wave and the stability of its radial pulsations is investigated using numerical analysis. The surrounding liquid is assumed to be viscous and incompressible.

In the investigation of processes of gas cavitation, propagation of perturbations in a gas–liquid medium, and other processes of similar type it is important to determine the effect of diffusion of the gas dissolved in the liquid on the dynamics of the cavity and its stability in an acoustic field.

The behavior of a gas bubble in a liquid was investigated in [1] taking account of the molecular diffusion of the gas at a constant external pressure. In [2] the solution was generalized to the case of variable pressure. In these studies the diffusion equation was solved disregarding the convective term introduced by the motion of the wall of the bubble. Experiments show that this is justified only for very slow changes of the pressure in the liquid [3]. In [4] the diffusion of the gas into the bubble in the field of an acoustic wave was determined taking account of the convective term in the diffuse boundary-layer approximation. The problem of diffusion of gas into a spherical cavity expanding at a constant rate is investigated in [5]. The effect of diffusion on the behavior of a cavitation gas bubble in an acoustic field has been investigated also in [6]. In [3-6] the problem is investigated by approximate methods. In [6], in particular, a quasi-stationary approach to the diffusion process is used, the inertial effects are taken into consideration, and the ordinary differential equation thus obtained for the bubble radius is integrated numerically. The dependence R(t) given in [6] agrees qualitatively with the results of the present article in the resonance case.

In the present article the evolution of the radius of a cavitation bubble and the stability of its radial pulsations are investigated using numerical integration and taking diffusion into consideration for different amplitudes and frequencies of the external acoustic field. The equation of nonstationary convective diffusion is considered:

$$\frac{\partial c}{\partial t} + \frac{R\dot{R}}{r^2}\cdot\frac{\partial c}{\partial r} = D\left(\frac{\partial^2 c}{\partial r^2} + \frac{2}{r}\cdot\frac{\partial c}{\partial r}\right). \tag{1}$$

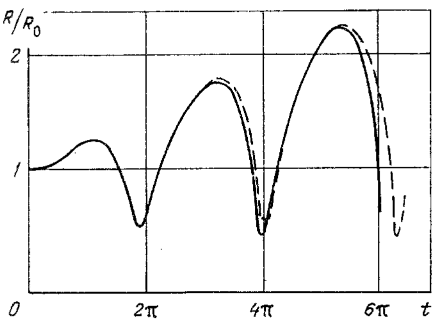

Fig. 1. The dependence of the radius of cavitation bubble R on the time t in the resonance case.

The change of the bubble radius R(t) is described by Nolting–Neppiras equation

$$R\ddot{R} + 3/2\,\dot{R}^2 + 1/\rho\left[P_0 - P_m \sin\omega t - p_g + \frac{2\sigma}{R} + \frac{4\mu\dot{R}}{R}\right] = 0. \tag{2}$$

The gas pressure p_g is determined from the condition of equality of the mass flux across the boundary of the cavity. According to Fick's law the mass flux into the bubble per unit time is

$$\frac{dm}{dt} = 4\pi R^2 D\left(\frac{\partial c}{\partial r}\right)_{r=R}. \tag{3}$$

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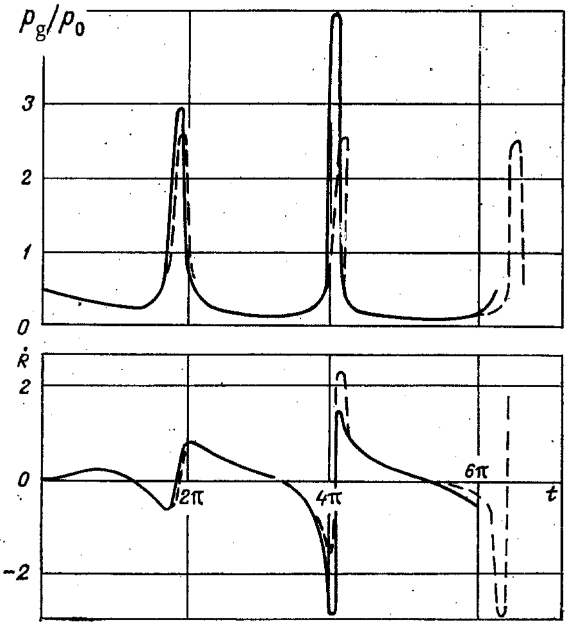

Fig. 2. Dependence of p_g and $\dot{R}$ on time t.

The mass of the gas in the bubble m_g is determined from the formula

$$m_g = 4/3\,\pi R^3 \rho_g(R). \tag{4}$$

Using the equation of state for the gas

$$p_g V_g = m_g \nu B T,$$

we obtain the density of the gas in the bubble

$$\rho_g(R) = m_g/V_g = \nu p_g/BT. \tag{5}$$

Substituting (5) into (4), finding dm_g/dt, and equating it to (3), we obtain the equation for R(t):

$$\dot{p}_g + 3\dot{R}p_g/R = \frac{3BTD}{\nu R}\left(\frac{\partial c}{\partial r}\right)_{r=R}. \tag{6}$$

Equations (1), (2), (6) constitute a closed system. The boundary and initial conditions are: $R(0) = R_0$; $\dot{R}(0) = 0$, $c(R, t) = c_s$; $c(\infty, t) = c(r, 0) = c_i$, $(r > R)$.

Let us consider the stability of the radial pulsations of a cavitation bubble. For this purpose we write the perturbed values of the radius $R'(t)$ of the bubble, gas pressure $p'(t)$, and concentration of the gas dissolved in the liquid $c'(r, t)$ in the form

$$R'(t) = R(t) + \xi(t), \tag{7}$$

$$p'_g(t) = p_g(t) + \eta(t), \tag{8}$$

$$c'(r, t) = c(r, t) + \zeta(r, t), \tag{9}$$

where ξ, η, ζ are, respectively, the perturbations of the radius, the gas pressure, and the concentration.

Substituting (7)-(9) into (1), (2), (6) and linearizing with respect to ξ, η, and ζ we obtain a system of equations for these quantities:

$$\frac{\partial \zeta}{\partial t} + \left(\frac{2R\dot{R}\xi}{r^2} + \frac{R^2}{r^2}\dot{\xi}\right)\frac{\partial c}{\partial r} + \frac{R^2\dot{R}}{r^2}\cdot\frac{\partial \zeta}{\partial r} = D\left(\frac{\partial^2 \zeta}{\partial r^2} + \frac{2}{r}\cdot\frac{\partial \zeta}{\partial r}\right), \tag{10}$$

$$\dot{\eta} + (3\dot{\xi}/R - 3\xi\dot{R}/R^2)\,p_r + 3\dot{R}\eta/R = -\xi/R^2\left(\frac{\partial c}{\partial r}\right)_{r=R} + \xi/R\left(\frac{\partial^2 c}{\partial r^2}\right)_{r=R} + \frac{1}{R}\left(\frac{\partial \zeta}{\partial r}\right)_{r=R}, \tag{11}$$

$$R\ddot{\xi} + \xi(\ddot{R} + 3\dot{R}) + 1/\rho\left(\frac{4\mu\dot{\xi}}{R} - \frac{4\mu\xi\dot{R}}{R^2} - \frac{2\sigma\xi}{R^2} - \eta\right) = 0. \tag{12}$$

Because of the complexity of Eqs. (1), (2), (6), (10)-(12) it is very difficult to find their analytical solution in the general case; therefore they were studied by numerical methods.

In (1) let us pass onto Lagrangian coordinates

$$y = \beta/3\,(r^3 - R^3), \quad \tau = \beta\int_0^t R\,dt.$$

Here β is a positive constant introduced for the sake of convenience of numerical analysis.

In terms of variables y, τ Eq. (1) becomes

$$R\,\frac{\partial c}{\partial \tau} = Dr\beta\left(\beta\,\frac{\partial^2 c}{\partial y^2} + 4\,\frac{\partial c}{\partial y}\right). \tag{13}$$

Equation (13) was approximated by an implicit three-layer finite-difference scheme of Crank–Nicolson type of the second order in both coordinates. The algebraic system of equations for the desired grid functions was solved by the trial-run method. The computation was done in the strip $0 \le y \le L$ and was terminated on reaching a prespecified value of τ. The computations were stable for almost all relations of the steps of numerical integration, which were chosen after a number of tests from the consideration of the best approximation of the difference scheme. Equation (10) for the perturbation of the concentration

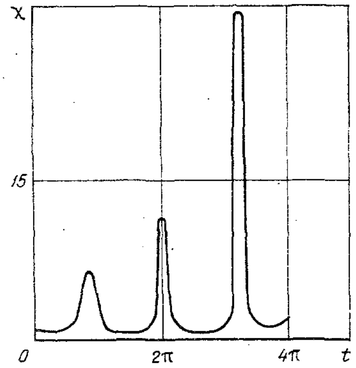

Fig. 3. Dependence of $\chi = \delta/R(t)$ on time t.

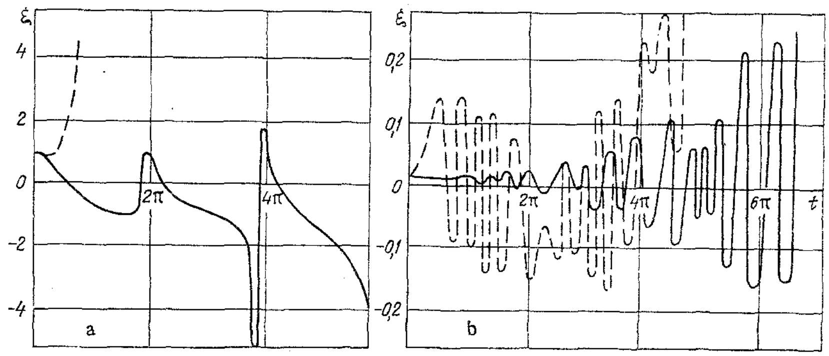

Fig. 4. Behavior of the perturbation $\xi(t)$ of the radial motion of cavitation bubble in the resonance (a) and preresonance (b) cases.

of the gas dissolved in the liquid was solved in the same way. Equations (2), (6), (11), (12) were integrated by the Runge–Kutta method. The characteristic frequency ω_0 of the bubble was calculated from the well-known formula [7]. Resonance ($\omega = \omega_0$), preresonance ($\omega = 0.2\omega_0$), and postresonance ($\omega = 5\omega_0$) conditions were considered. Everywhere the values $R_0 = 10^{-4}$ cm, $P_0 = 1$ atm were used. The amplitude of the external acoustic field was 0.5 atm in the resonance and preresonance cases and 100 atm in the postresonance case; the value of β was $\beta = 10$.

On the basis of the characteristics of the diffusion process it can be assumed that it retards the process of evolution of the bubble radius and increases the minimum radius R_{min} of the bubble in the collapse phase. This is due to the fact that diffusion effects the pressure balance at the boundary of the cavity. During the compression of the bubble the pressure inside it increases. Under the effect of diffusion processes a part of the gas inside the bubble crosses over into the liquid, which leads to a decrease of the pressure in the cavity. Since the radius of the bubble is inversely proportional to p_g, R_{min} increases. The characteristic dependence of the radius on t is shown in Fig. 1 for the resonance case without considering viscosity and surface tension. The dashed line shows the behavior of the radius with diffusion taken into consideration. The behavior of p_g and R is shown in Fig. 2 for $\omega = \omega_0$. Here and in subsequent figures the dashes show the behavior of the corresponding quantities with diffusion taken into consideration.

We note that in course of time the thickness δ of the diffuse boundary layer near the surface of the bubble increases, attaining its maximum value in the compression phase. The dependence of the quantity $\chi = \delta/R(t)$ on t is shown in Fig. 3. Here for δ we chose such a distance from the surface of the cavitation bubble, at which the concentration reached 50% of its value at infinity.

The numerical analysis presented above shows that the diffusion processes make the radial pulsations of the bubble less stable. The instability begins in the expansion phase when the pressure inside the bubble increases. The behavior of the perturbation $\xi(t)$ of the radial motion of the cavitation bubble is shown in Fig. 4 for the resonance (Fig. 4a) and preresonance (Fig. 4b) cases.

The above discussion elucidates certain characteristics of the effect of diffusion on the evolution of gas bubbles in the field of an acoustic wave.

NOTATION

- c is the concentration of dissolved gas;
- R is the radius of the bubble;
- D is the diffusion coefficient;
- r is the radial coordinate;
- t is the time;
- ρ is the liquid density;
- ω_0 is the characteristic frequency of the bubble;
- ω is the frequency of the external field;
- P_m is the amplitude of the external field;
- P_0 is the unperturbed pressure in the liquid;
- p_g is the gas pressure in the bubble;
- σ is the coefficient of surface tension;
- μ is the coefficient of viscosity of the liquid;

m_g is the mass of the gas in the bubble;
ν is the molecular weight;
B is the universal gas constant;
T is the gas temperature;
V_g is the volume of the gas in the bubble;
ξ is the perturbation of the radius;
η is the perturbation of the pressure;
ζ is the perturbation of the concentration.

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